

On the Quantum Chromodynamics of a Massive Vector Field in the Adjoint Representation

Alfonso R. Zerwekh*

*Departamento de Física and
Centro Científico-Tecnológico de Valparaíso
Universidad Técnica Federico Santa María
Casilla 110-V, Valparaíso, Chile*

Abstract

In this paper, I explore the construction of the QCD of a massive color-octet vector field without introducing higher structures like extended gauge symmetries, extra dimensions or scalar fields. I show that gauge invariance is not enough to constrain the couplings. Nevertheless the requirement of unitarity fixes the values of the coupling constants, which otherwise would be arbitrary, and opens a new discrete symmetry. On the other hand, a judicious definition of the gauge fixing terms modify the propagator of the massive field making it well-behaved in the ultra-violet limit. The new discrete symmetry also makes appear a dark matter candidate. Additionally it is shown that the model can be formulated as a Yang-Mills theory with two connections. I discuss the phenomenological implications of the model and speculate about its renormalizability.

1 Introduction

Many extensions of the Standard Model, such as non-minimal Technicolor [1, 2, 4, 5, 3], Extra-dimensions [6], Top-color [7, 8] and Chiral-color [9, 10, 11, 12], predict the existence of massive color-octet spin-1 particles which I will collectively call “colorons”. In principle, it is expected that, if a coloron exists in the appropriated mass range, it should be copiously produced at hadron colliders such as the Tevatron or the LHC [5, 13]. Indeed, some renewed interest on this kind of particles has arisen [14, 15] because some sort of color-octet spin-1 resonance may be the origin of the large $t\bar{t}$ forward-backward asymmetry measured by CDF [16, 17] and D0 [18].

From the phenomenological point of view, it is convenient, given the large variety of models predicting colorons, to find an effective model independent description which can grasp their essential features. This is the origin, for example, of the deconstruction idea [19, 20]: the initial intension was to describe

*alfonso.zerwekh@usm.cl

the Kaluza-Klein excitations of the gluon independently of the details of the underlying extra-dimensional theory. A similar problem, but in the context of non-minimal Technicolor, motivated another effective description [3, 5] based on the observed low-energy symmetries. This phenomenological analysis produced two important results: it was shown that the s -channel production of a single coloron is plagued by theoretical uncertainties [5] and it was argued that the coloron pair production would be almost model independent [5, 13], being determined exclusively by QCD gauge invariance.

Interestingly, when one tries to make the quantum chromodynamics of a coloron, one finds that there are at least two different, although gauge equivalent, formulations [3, 21]. Both of them lead to theories with a bad ultraviolet behavior due to the presence of the massive spin-1 field. This is a quite frustrating situation because we know how to construct consistent and renormalizable quantum field theories with scalars or fermions (massive or not) as matter fields but things seem to be very different when a massive spin-1 field is considered.

In this work, I revisit the construction of a gauge theory for the coloron and I examine the possibility that such a theory be consistent with renormalizability and unitarity without introducing neither scalar fields nor higher structures such as extra-dimensions or extended gauge symmetries. For this purpose, I organized this paper in the following way. In section 2, I describe the construction of a general classical gauge theory with a massive spin-one field in the adjoint representation. In section 3, I move to the quantum version of the theory, paying special attention to the gauge fixing and the ghost terms, and implementing the BRST symmetry. Section 4 is devoted to study the constraints imposed by requiring that perturbative unitarity of the S-matrix holds at tree-level. Finally, I summarize my conclusions in section 5.

2 A Gauge Theory for a Massive Vector Field

2.1 Global Symmetry

Usually, the starting point for studying the physical properties of a massive spin-one field is the Proca Lagrangian. So, let's consider a generalization of the Proca theory for a non-Abelian *global* continuous symmetry:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2}M^2 A_\mu^a A^{a\mu} - A_\mu^a J^{a\mu} (+\mathcal{L}_{int}) \quad (1)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$ and A_μ^a transforms homogeneously under the global symmetry ($A_\mu \rightarrow U^\dagger A_\mu U$). I have included an external source ($J^{a\mu}$) which is supposed to be a conserved current. Additionally, all the other invariant terms that can be constructed with A_μ and $\partial_\mu A_\nu$ can be eventually included in \mathcal{L}_{int} , but they will not be relevant for the present analysis and, for simplicity, will not be taken explicitly into account. As it is well known, the field equation obtained from Lagrangian (1) can be written as:

$$\partial_\rho F^{a\rho\nu} + M^2 A^{a\nu} = J^{a\nu} \quad (2)$$

and the anti-symmetry of $F_{\mu\nu}$ automatically implies the Lorenz condition:

$$\partial_\mu A^{a\mu} = 0 \quad (3)$$

Let me recall, for completeness, that equation (3) eliminate one degree of freedom from A_μ . Naturally, the remaining degrees of freedom correspond to the three polarization states of a massive spin-one particle.

Unfortunately, when a quantum theory is constructed from Lagrangian (1), it leads to the following propagator:

$$\Delta_{\mu\nu} = \frac{-i}{q^2 - M^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2} \right) \quad (4)$$

which spoils the renormalizability of the theory due to the bad ultraviolet behavior of the last term. Interestingly, the form of this propagator, and its ultraviolet divergency, can be traced to the anti-symmetric structure of $F_{\mu\nu}$. So, we can raise the reasonable question of whether an anti-symmetric $F_{\mu\nu}$ is essential to our theory, or not. In other words, is there a fundamental principle that compels us to include $\partial_\mu A_\nu \partial^\mu A^\nu$ and $\partial_\mu A_\nu \partial^\nu A^\mu$ in the Lagrangian with the same weight? Certainly, the answer is negative. In gauge theory, the anti-symmetry of $F_{\mu\nu}$ is dictated by the gauge principle since its necessary to cancel the inhomogeneous part of the transformation of the gauge field, but this is not the case here because in our construction A_μ transforms homogeneously. Consequently, it is possible to write down a more general Lagrangian:

$$\mathcal{L} = -\frac{1}{2} \partial_\mu A_\nu^a \partial^\mu A^{a\nu} + \frac{(1+a)}{2} \partial_\mu A_\nu^a \partial^\nu A^{a\mu} + \frac{1}{2} M^2 A_\mu^a A^{a\mu} - A_\mu^a J^{a\mu} (+\mathcal{L}_{int}) \quad (5)$$

Of course, the Proca Lagrangian is recovered for $a = 0$. This time, the field equation is:

$$\partial^2 A_\mu - a \partial_\mu \partial^\sigma A_\sigma + M^2 A_\mu = J_\mu \quad (6)$$

where I have dropped the group index. Differentiating (6), it follows that a generalized Lorenz condition is satisfied:

$$\partial_\mu A^\mu = f(x) \quad (7)$$

where $f(x)$ is a solution of the following equation:

$$\left[\partial^2 - \frac{M^2}{a} \right] f(x) = 0 \quad (8)$$

Of course, $f(x) = 0$ is a solution of (8) and the usual Lorenz condition can be used.

A more important consequence, however, is the fact that the propagator obtained from (5) can be written in following way:

$$\Delta_{\mu\nu} = \frac{-i}{q^2 - M^2} \left(g_{\mu\nu} - \frac{(1+a) q_\mu q_\nu}{a q^2 - M^2} \right) \quad (9)$$

Notice that this new propagator has the same form of the propagator of a massive gauge boson in the context of spontaneously broken gauge symmetries, with a playing the role of a gauge parameter. This modified propagator behaves adequately in the ultraviolet limit.

In the Abelian case, the theoretical construction presented so far is similar to the one resulting from the Stueckelberg theory when the compensating scalar field is gauged away. Indeed, equations (6) and (8) are formally equal to those obtained from the Stueckelberg Lagrangian [22]. It is worth to recall that the Stueckelberg formalism makes the theory of a massive photon renormalizable. For this reason, Lagrangian (5) seems to be a good starting point in the attempt of making a consistent theory for the coloron.

2.2 Local Symmetry

Evidently, the most direct way to turn the previous construction into a local gauge theory is to take Lagrangian (5), replace partial derivatives by covariant ones, include a Yang-Mills term for the gluon, include in \mathcal{L}_{int} all the gauge invariant and renormalizable terms we can form with A_μ , $D_\mu A_\nu$ and $G_{\mu\nu}$ (strength field of gluons) with arbitrary coefficients, without forgetting a kinetic mixing term $G_{\mu\nu}F^{\mu\nu}$.

I will not follow this path, however. For reasons that will become apparent in the next section, it is convenient to consider a different, but completely equivalent approach. We will work in a picture where there are two fields (namely, $A_{1\mu}^a$ and $A_{2\mu}^a$) which transform as gauge fields under the *same* gauge group:

$$A_{i\mu} \rightarrow U A_{i\mu} U^{-1} - \frac{1}{g_i} (\partial_\mu U) U^{-1} \quad (i = 1, 2) \quad (10)$$

where $A_{i\mu} \equiv A_{i\mu}^a T^a$ and T^a represents the generators of the algebra.

With these fields we can define two different covariant derivatives:

$$D_{j\mu} \equiv \partial_\mu - i g_j A_{j\mu} \quad (\text{no summation in } j) \quad (11)$$

and with them we can appropriately define strength fields:

$$F_{j\mu\nu} \equiv \frac{i}{g_j} [D_{j\mu}, D_{j\nu}] \quad (12)$$

Notice that any object of the form $D_\mu = \chi D_{1\mu} + (1 - \chi) D_{2\mu}$, where χ is an arbitrary parameter, behaves as a good covariant derivative. Consequently, the general covariant derivative used in the construction of the Lagrangian is:

$$D_\mu = \partial_\mu - i \chi g_1 A_{1\mu} - i (1 - \chi) g_2 A_{2\mu} \quad (13)$$

A key remark, based on (10), is that since both fields transform like gauge fields under the same group, the special combination $\mathcal{V}_\mu \equiv g_1 A_{1\mu} - g_2 A_{2\mu}$ transforms homogeneously.

With all these tools at hand, we can write down the following Lagrangian:

$$\begin{aligned}
\mathcal{L}_{2CYM} = & -\frac{1}{2}Tr[F_{1\mu\nu}F_1^{\mu\nu}] - \frac{1}{2}Tr[F_{2\mu\nu}F_2^{\mu\nu}] \\
& + \frac{M^2}{g_1^2 + g_2^2}Tr[(g_1A_{1\mu} - g_2A_{2\mu})^2] \\
& + \frac{a}{g_1^2 + g_2^2}Tr[(g_1D_\mu A_{1\nu} - g_2D_\mu A_{2\nu})(g_1D^\nu A_1^\mu - g_2D^\nu A_2^\mu)] \\
& + \mathcal{L}_{NM}
\end{aligned} \tag{14}$$

where “2CYM” stands for “Two-Connection-Yang-Mills” theory. Notice that the third term originates a non-diagonal mass matrix. On the other hand, the fourth term corresponds to the previously discussed Stueckelberg term. Finally, \mathcal{L}_{NM} (where “NM” stands for “Non-Minimal”) include all the gauge invariant terms that can be constructed using V_μ , $D_\mu V_\nu$, $F_{1\mu\nu}$ and $F_{2\mu\nu}$.

When we diagonalize the mass matrix, we find that the physical fields (mass eigenstates) are:

$$G_\mu = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}A_{1\mu} + \frac{g_1}{\sqrt{g_1^2 + g_2^2}}A_{2\mu} \tag{15}$$

$$V_\mu = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}A_{1\mu} - \frac{g_2}{\sqrt{g_1^2 + g_2^2}}A_{2\mu} \tag{16}$$

It is straightforward to prove that now G_μ is massless and transforms like a connection while V_μ is the massive state and transforms homogeneously. Hence, it is natural to identify G_μ as the gluon field and V_μ as the coloron. In terms of the physical fields, Lagrangian (14) takes the following form :

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}Tr\{G_{\mu\nu}G^{\mu\nu}\} - Tr\{D_\mu V_\nu D^\mu V^\nu\} + (1+a)Tr\{D_\mu V_\nu D^\nu V^\mu\} \\
& + a_{11}Tr\{D_\mu V_\nu V^\mu V^\nu\} + a_{12}Tr\{D_\mu V_\nu V^\nu V^\mu\} \\
& + a_{21}Tr\{V_\mu V_\nu V^\mu V^\nu\} + a_{22}Tr\{V_\mu V_\nu V^\nu V^\mu\} \\
& + a_3Tr\{G_{\mu\nu}[V^\mu, V^\nu]\} + M^2Tr\{V_\nu V^\nu\}
\end{aligned} \tag{17}$$

where arbitrary constant appears because of the contributions from \mathcal{L}_{NM} . After the diagonalization of the mass matrix, the covariant derivative (13) becomes:

$$D_\mu = \partial_\mu - igG_\mu - i\frac{[\chi(g_1^2 + g_2^2) - g_2^2]}{\sqrt{g_1^2 + g_2^2}}V_\mu \tag{18}$$

where

$$g \equiv \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \tag{19}$$

is the usual QCD coupling constant. Interestingly, only the third term in the covariant derivative depends on χ . If we put (18) back into the Lagrangian (17) and expand it, we will see that the last term of (18) will generate terms which

are already present in the Lagrangian with arbitrary constants, so the new term can be absorbed. That means that it is enough, in the context of (17), to use the usual definition of covariant derivative.

$$D_\mu = \partial_\mu - igG_\mu \quad (20)$$

Notice that the third term in (17) includes the implementation of the Stueckelberg trick and, in the quantum version of the theory developed so far, the propagator of the coloron would be the one shown in (9). Nevertheless, the same term modifies the GVV and the $GGVV$ vertex (the last one is also modified by the term proportional to a_3). This fact is important because it means that it is not guaranteed that, in the general case, a massive color-octet spin-one particle interacts with gluons with a typical QCD strength as it is commonly believed.

3 Quantum Theory: BRST Symmetry

Strictly speaking, what we have done so far is to develop a classical theory. If we want to quantize the theory using the path integral method, it is necessary to add gauge fixing terms and ghost fields as dictated by the Fadeev-Popov procedure. Fortunately, this is an easy task in the version of the model developed in the previous section. A good starting point is Lagrangian (14). Because we have two gauge-like fields, all we need to do is to duplicate the standard Fadeev-Popov prescription and add to (14) the following Lagrangian:

$$\begin{aligned} \mathcal{L}_{GF} = & \frac{1}{2}\xi_1 B_1^a B_1^a - B_1^a \partial^\mu A_{1\mu}^a + \bar{c}_1^a \partial^\mu D_{1\mu}^{ab} c^b \\ & + \frac{1}{2}\xi_2 B_2^a B_2^a - B_2^a \partial^\mu A_{2\mu}^a + \bar{c}_2^a \partial^\mu D_{2\mu}^{ab} c^b \end{aligned} \quad (21)$$

where, as usual, B_1^a and B_2^a are auxiliary fields, c , \bar{c}_1 and \bar{c}_2 are ghost and anti-ghost fields and ξ_1 and ξ_2 are gauge parameters. In order to avoid the introduction of kinetic mixing terms in the physical basis (because I expect that in the basis formed by G_μ and V_μ everything is diagonal), we chose $\xi_1 = \xi_2 = \xi$. Of course, by construction, the whole Lagrangian (that is, (14) + (21)) is invariant under the BRST transformations:

$$\delta_B A_{i\mu}^a = \frac{1}{g_i} D_{i\mu}^{ab} c^b \quad (22)$$

$$\delta_B c^a = -\frac{1}{2} f^{abc} c^b c^c \quad (23)$$

$$\delta_B \bar{c}_i^a = B_i^a \quad (24)$$

$$\delta_B B_i^a = 0 \quad (25)$$

In the physical basis, Lagrangian (21) takes the form:

$$\begin{aligned} \mathcal{L}_{GF} = & -\frac{1}{2\xi} (\partial^\mu G_\mu^a)^2 - \frac{1}{2\xi} (\partial^\mu V_\mu^a)^2 + \bar{c}^a \partial^\mu D_\mu^{ab} c^b \\ & + \alpha f^{abc} (\partial^\mu \bar{c}^a) V_\mu^c c^b + \beta f^{abc} (\partial^\mu \bar{\eta}^a) V_\mu^c c^b \end{aligned} \quad (26)$$

where we have already eliminated the auxiliary fields. Here α and β are some combinations of the original coupling constants g_1 and g_2 but their exact expressions are not important for our purposes. On the other hand, \bar{c} and $\bar{\eta}$ are related to the previous anti-ghosts by the following definitions:

$$\bar{c} \equiv \bar{c}_1 + \bar{c}_2 \quad (27)$$

$$\bar{\eta} \equiv \bar{c}_2 - \bar{c}_1 \quad (28)$$

An important characteristic of Lagrangian (26) is that $\bar{\eta}$ doesn't have a kinetic term and hence its equation of motion is only a constraint:

$$f^{abc} \partial^\mu (V_\mu^c c^b) = 0 \quad (29)$$

Interestingly, this is exactly the kind of constraint needed to implement the Lorenz condition for a massive field transforming homogeneously under the symmetry group. Putting (29) back in the Lagrangian, we find:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} (\partial^\mu G_\mu^a)^2 - \frac{1}{2\xi} (\partial^\mu V_\mu^a)^2 + \bar{c}^a \partial^\mu D_\mu^{ab} c^b \quad (30)$$

So finally, we have the correct gauge fixing and ghost terms of our model. The most important consequence of this procedure is that the second term of (30) contributes to the propagator of V_μ . Indeed, we can write now the correct propagator for the gluon and the coloron in the complete theory:

$$\Delta_G = \frac{-i\delta^{ab}}{q^2} \left(g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2} \right) \quad (31)$$

$$\Delta_V = \frac{-i\delta^{ab}}{q^2 - M^2} \left(g^{\mu\nu} + (\xi + \xi a - 1) \frac{q^\mu q^\nu}{(1 - \xi a)q^2 - \xi M^2} \right) \quad (32)$$

4 Unitarity

4.1 Unitarity Constrains

Hitherto, we have constructed a general quantum gauge theory of the coloron with operators of dimension four or less. It can be seen as a good starting point for an effective theory and its main consequence is that the coupling of the coloron to gluon may sensible deviate from the general expectation. However, I would like to recall that the aim of this work is to explore the construction of a coloron theory which can be well behaved in the ultraviolet limit and, eventually, renormalizable. In this sense, it is necessary to compel the theory to preserve the perturbative unitarity of the S-matrix. For this purpose, I compute the amplitude for the processes $V_L V_L \rightarrow V_L V_L$ and $GG \rightarrow V_L V_L$ (where G is the gluon and V_L is the longitudinally polarized coloron) at tree-level and I impose the condition that the terms which are divergent in the ultraviolet limit

($s/M^2 \rightarrow \infty$) vanish. In these calculations, I will use a simplified version of (17):

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}Tr\{G_{\mu\nu}G^{\mu\nu}\} - Tr\{D_\mu V_\nu D^\mu V^\nu\} + (1+a)Tr\{D_\mu V_\nu D^\nu V^\mu\} \\
& + a_1 Tr\{(D_\mu V_\nu - D_\nu V_\mu)[V^\mu, V^\nu]\} \\
& + a_2 Tr\{[V_\mu, V_\nu][V^\mu, V^\nu]\} \\
& + a_3 Tr\{G_{\mu\nu}[V^\mu, V^\nu]\} + M^2 Tr\{V_\nu V^\nu\} \\
& + \mathcal{L}_{GF}
\end{aligned} \tag{33}$$

This simplification is well motivated, however, because the terms in (17) containing three and four V fields would give origin in the amplitude to terms proportional to the d^{abc} (the completely symmetric constants of the group) which should cancel among themselves. The best way to assure this cancellation is organizing the V self-interactions in terms of the commutators. The divergent part of the amplitude for the $V_L V_L \rightarrow V_L V_L$ scattering can be written as:

$$\begin{aligned}
\mathcal{M} = & \frac{(a_2 + g^2 + a_1^2)(t^2 - 2tu - 2u^2)(t+u)^2 itu}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^4} f^{abe} f^{cde} \\
& + \frac{(a_2 + g^2 + a_1^2)(t^2 + 4tu + u^2)(t+u)^2 itu}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^4} f^{ace} f^{bde} \\
& - \frac{(a_2 + g^2)(t^4 + 11t^3u - 23t^2u^2 - 28tu^3 - 2u^4)(t+u)i}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^2} f^{abe} f^{cde} \\
& - \frac{6i(t+u)^2 ag^2 t^2 u}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^2} f^{abe} f^{cde} \\
& - \frac{(t^4 + 14t^3u - 20t^2u^2 - 28tu^3 - 2u^4)a_1^2(t+u)i}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^2} f^{abe} f^{cde} \\
& - \frac{(t^4 + 14t^3u + 40t^2u^2 + 14tu^3 + u^4)a_1^2(t+u)i}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^2} f^{ace} f^{bde} \\
& + \frac{6i(t+u)^3 ag^2 tu}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^2} f^{ace} f^{bde} \\
& - \frac{(a_2 + g^2)(t^4 + 17t^3u + 46t^2u^2 + 17tu^3 + u^4)(t+u)i}{4s(M^2 - s)(M^2 - t)(M^2 - u)M^2} f^{ace} f^{bde} \\
& + \mathcal{O}\left(\frac{M^2}{s}\right)
\end{aligned} \tag{34}$$

It can be easily seen that, in order to cancel the problematic terms, it is enough to satisfy the following conditions:

$$a = 0 \tag{35}$$

$$a_1 = 0 \tag{36}$$

$$a_2 = -g^2 \tag{37}$$

Indeed, with this election of parameters, the amplitud for the $V_L V_L \rightarrow V_L V_L$ scattering completely vanishes. Surprisingly, the previous conditions avoid the presence of the Stueckerberg term and forbid the coloron triple vertex.

In a similar way, imposing unitarity to the $GG \rightarrow V_L V_L$ scattering amplitude, we get an additional condition:

$$a_3 = -g \quad (38)$$

Consequently, taken into account the restrictions due to unitarity, the Lagrangian takes the simple form:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}Tr\{G_{\mu\nu}G^{\mu\nu}\} - Tr\{D_\mu V_\nu D^\mu V^\nu\} + Tr\{D_\mu V_\nu D^\nu V^\mu\} \\ & -g^2 Tr\{[V_\mu, V_\nu][V^\mu, V^\nu]\} \\ & -gTr\{G_{\mu\nu}[V^\mu, V^\nu]\} + M^2 Tr\{V_\nu V^\nu\} \\ & -\frac{1}{2\xi}(\partial^\mu G_\mu^a)^2 - \frac{1}{2\xi}(\partial^\mu V_\mu^a)^2 + \bar{c}^a \partial^\mu D_\mu^{ab} c^b \end{aligned} \quad (39)$$

and the propagators are:

$$\Delta_G = \frac{-i\delta^{ab}}{q^2} \left(g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2} \right) \quad (40)$$

$$\Delta_V = \frac{-i\delta^{ab}}{q^2 - M^2} \left(g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2 - \xi M^2} \right) \quad (41)$$

Notice that the coloron propagator is the same one we would have obtained for a massive spin-one field in a gauge theory with spontaneous symmetry breaking.

4.2 Consequences of Unitarity

Interestingly, Lagrangian (39) posses a new Z_2 symmetry under which V is odd and G is even. This discrete symmetry makes the coloron to be stable. Hence, an unexpected consequence of unitarity is that the coloron, conveniently dressed by gluons, will form a new kind of stable hadron that can be a cold dark matter candidate. Another implication of the Z_2 symmetry is that this kind of coloron cannot be resonantly produced at a collider. The easiest way to create it, is pair production. Naturally, the produced colorons will hadronize producing two jets. Because of the huge background for two jets at a hadron collider and the absence of any distinctive kinematical structure, I expect that the observation of this kind of colorons at the LHC would be very challenging.

In the “Two-Connections” picture, on the other hand, the Z_2 symmetry translates as a symmetry of the Lagrangian under the interchange of the two connections ($A_1 \leftrightarrow A_2$). Imposing this symmetry to Lagrangian (14), we obtain that the coupling constants must satisfy the condition $g_1 = g_2 = \sqrt{2}g$ and the

Lagrangian (including the gauge fixing and ghost sectors) takes the simple form:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}\text{Tr}[F_{1\mu\nu}F_1^{\mu\nu}] - \frac{1}{2}\text{Tr}[F_{2\mu\nu}F_2^{\mu\nu}] + \frac{M^2}{2}\text{Tr}[(A_{1\mu} - A_{2\mu})^2] \\ & - \frac{1}{2\xi}(\partial^\mu A_{1\mu}^a)^2 - \frac{1}{2\xi}(\partial^\mu A_{2\mu}^a)^2 + \frac{\bar{c}^a}{2}\partial^\mu D_{1\mu}^{ab}c^b + \frac{\bar{c}^a}{2}\partial^\mu D_{2\mu}^{ab}c^b \end{aligned} \quad (42)$$

Obviously, Lagrangian (39) is automatically obtained from (42) after the diagonalization of the mass matrix.

5 Summary and Conclusions

Finally, we have arrived to our goal and now it is time to recapitulate our main results. First, we studied the construction of a general local gauge theory for the coloron (with operators of dimension up to 4). We saw that it is plagued of undetermined coupling constants, but, nevertheless, this general theory can be a good starting point for effective models. A direct consequence of this degree of arbitrariness is that the expectation of the coloron interacting with gluon with “typical QCD intensity” is not guaranteed.

In a second step, we were able to construct a particular gauge theory for the coloron which is BRST invariant, consistent with perturbative unitarity and produces propagators with acceptable ultraviolet behavior. Additionally, the conditions imposed by unitarity are protected by the emergence of a discrete symmetry. For all these reasons and from the point of view of power counting, we can expect that the theory should be renormalizable. However, a formal proof must still be provided.

This model is phenomenologically interesting since it constitutes a simple extension of QCD and provide an eventual cold dark matter candidate. However, an observation of this kind of coloron at the LHC seems to be difficult.

From a theoretical point of view, it is interesting that such a well behaved theory can be formulated as a simple Yang-Mills theory with two connections. In some sense, this is the quantum field theory analog of the long ago proposed bigravity extensions of General Relativity [23] which have recently called the attention of the gravitation community [24, 25].

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